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T.B.C. : ASGT-B-GNL

Test Booklet Series

TEST BOOKLET
MATHEMATICS

A

Time Allowed : Two Hours and Thirty Minutes

Maximum Marks : 300

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET ***DOES NOT*** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. **Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Test Booklet Series Code A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the Answer Sheet liable for rejection.**
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. ***DO NOT*** write ***anything else*** on the Test Booklet.
4. This Test Booklet contains **120** items (questions). Each item is printed both in **Hindi** and **English**. Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ***ONLY ONE*** response for each item.
5. You have to mark all your responses ***ONLY*** on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. ***All*** items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator ***only the Answer Sheet***. You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong Answers :**
    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.
    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.
    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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ध्यान दें : अनुदेशों का हिन्दी रूपान्तर इस पुस्तिका के मुख पृष्ठ पर छपा है ।

1. If ω is a non-real cube root of 1, then what is the value of $\left|\frac{1-\omega}{\omega+\omega^2}\right|$?

(a) $\sqrt{3}$

(b) $\sqrt{2}$

(c) 1

(d) $\frac{4}{\sqrt{3}}$

2. What is the number of 6-digit numbers that can be formed only by using 0, 1, 2, 3, 4 and 5 (each once); and divisible by 6 ?

(a) 96

(b) 120

(c) 192

(d) 312

3. What is the binary number equivalent to decimal number 1011 ?

(a) 1011

(b) 111011

(c) 11111001

(d) 111110011

4. Let A be a matrix of order 3×3 and $|A| = 4$. If $|2 \text{ adj}(3A)| = 2^{\alpha}3^{\beta}$, then what is the value of $(\alpha+\beta)$?

(a) 12

(b) 13

(c) 17

(d) 24

5. If α and β are the distinct roots of equation $x^2 - x + 1 = 0$, then what is the value of $\left|\frac{\alpha^{100}+\beta^{100}}{\alpha^{100}-\beta^{100}}\right|$?

(a) $\sqrt{3}$

(b) $\sqrt{2}$

(c) 1

(d) $\frac{1}{\sqrt{3}}$

6. Let A and B be symmetric matrices of same order, then which one of the following is correct regarding $(AB - BA)$?

1. Its diagonal entries are equal but nonzero

2. The sum of its non-diagonal entries is zero

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

7. Consider the following statements in respect of square matrices A, B, C each of same order n :

1. $AB = AC \Rightarrow B = C$ if A is non-singular
2. If $BX = CX$ for every column matrix X having n rows then $B = C$

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

8. The system of linear equations $x+2y+z=4$, $2x+4y+2z=8$ and $3x+6y+3z=10$ has

(a) a unique solution

(b) infinite many solutions

(c) no solution

(d) exactly three solutions

9. Let $AX = B$ be a system of 3 linear equations with 3-unknowns. Let X_1 and X_2 be its two distinct solutions. If the combination $aX_1 + bX_2$ is a solution of $AX = B$; where a, b are real numbers, then which one of the following is correct ?

(a) $a = b$

(b) $a + b = 1$

(c) $a + b = 0$

(d) $a - b = 1$

10. What is the sum of the roots of the equation $\begin{vmatrix} 0 & x-a & x-b \\ 0 & 0 & x-c \\ x+b & x+c & 1 \end{vmatrix} = 0$?

(a) $a + b + c$

(b) $a - b + c$

(c) $a + b - c$

(d) $a - b - c$

11. If $2 - i\sqrt{3}$ where $i = \sqrt{-1}$ is a root of the equation $x^2 + ax + b = 0$, then what is the value of $(a + b)$?

(a) –11

(b) –3

(c) 0

(d) 3

12. If $z = \dfrac{1+i\sqrt{3}}{1-i\sqrt{3}}$ where $i = \sqrt{-1}$, then what is the argument of z ?

(a) $\dfrac{\pi}{3}$

(b) $\dfrac{2\pi}{3}$

(c) $\dfrac{4\pi}{3}$

(d) $\dfrac{5\pi}{6}$

13. If a, b, c are in AP, then what is

$$\begin{vmatrix} x+1 & x+2 & x+3 \\ x+2 & x+3 & x+4 \\ x+a & x+b & x+3 \end{vmatrix}$$ equal to ?

(a) -1

(b) 0

(c) 1

(d) 2

14. If $\log_x a$, a^x and $\log_b x$ are in GP, then what is x equal to ?

(a) $\log_a(\log_b a)$

(b) $\log_b(\log_a b)$

(c) $\frac{\log_a(\log_b a)}{2}$

(d) $\frac{\log_b(\log_a b)}{2}$

15. If $2^{\frac{1}{c}}$, $2^{\frac{b}{ac}}$, $2^{\frac{1}{a}}$ are in GP, then which one of the following is correct ?

(a) a, b, c are in AP

(b) a, b, c are in GP

(c) a, b, c are in HP

(d) ab, bc, ca are in AP

16. The first and the second terms of an AP are $\frac{5}{2}$ and $\frac{23}{12}$ respectively. If n^{th} term is the largest negative term, what is the value of n ?

(a) 5

(b) 6

(c) 7

(d) n cannot be determined

17. For how many integral values of k, the equation $x^2 - 4x + k = 0$, where k is an integer has real roots and both of them lie in the interval (0, 5) ?

(a) 3

(b) 4

(c) 5

(d) 6

18. In an AP, the first term is x and the sum of the first n terms is zero. What is the sum of next m terms ?

(a) $\frac{mx(m+n)}{n-1}$

(b) $\frac{mx(m+n)}{1-n}$

(c) $\frac{nx(m+n)}{m-1}$

(d) $\frac{nx(m+n)}{1-m}$

19. Consider the following statements :

1. (25)! + 1 is divisible by 26
2. (6)! + 1 is divisible by 7

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

20. If z is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then what is $|z|$ equal to ?

(a) $\frac{1}{2}$

(b) $\frac{2}{3}$

(c) 1

(d) 2

21. How many real numbers satisfy the equation $|x-4|+|x-7| = 15$?

(a) Only one

(b) Only two

(c) Only three

(d) Infinitely many

22. A mapping $f: A \to B$ defined as $f(x) = \frac{2x+3}{3x+5}$, $x \in A$. If f is to be onto, then what are A and B equal to ?

(a) $A = R\setminus\{-\frac{5}{3}\}$ and $B = R\setminus\{-\frac{2}{3}\}$

(b) $A = R$ and $B = R\setminus\{-\frac{5}{3}\}$

(c) $A = R\setminus\{-\frac{3}{2}\}$ and $B = R\setminus\{0\}$

(d) $A = R\setminus\{-\frac{5}{3}\}$ and $B = R\setminus\{\frac{2}{3}\}$

23. α and β are distinct real roots of the quadratic equation $x^2 + ax + b = 0$. Which of the following statements is/are sufficient to find α ?

1. $\alpha+\beta = 0$, $\alpha^2+\beta^2 = 2$

2. $\alpha\beta^2 = -1$, $a = 0$

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

24. If the sixth term in the binomial expansion of $\left(x^{-\frac{8}{3}} + x^2 \log_{10} x\right)^8$ is 5600, then what is the value of x ?

(a) 6

(b) 8

(c) 9

(d) 10

25. How many terms are there in the expansion of $(3x-y)^4(x+3y)^4$?

(a) 9

(b) 12

(c) 15

(d) 17

26. p, q, r and s are in AP such that $p+s=8$ and $qr=15$. What is the difference between largest and smallest numbers ?

(a) 6

(b) 5

(c) 4

(d) 3

27. Consider the following statements for a fixed natural number n :

1. $C(n, r)$ is greatest if $n=2r$
2. $C(n, r)$ is greatest if $n=2r-1$ and $n=2r+1$

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

28. m parallel lines cut n parallel lines giving rise to 60 parallelograms. What is the value of $(m+n)$?

(a) 6

(b) 7

(c) 8

(d) 9

29. Let x be the number of permutations of the word 'PERMUTATIONS' and y be the number of permutations of the word 'COMBINATIONS'. Which one of the following is correct ?

(a) $x=y$

(b) $y=2x$

(c) $x=4y$

(d) $y=4x$

30. 5-digit numbers are formed using the digits 0, 1, 2, 4, 5 without repetition. What is the percentage of numbers which are greater than 50,000 ?

(a) 20%

(b) 25%

(c) $\frac{100}{3}\%$

(d) $\frac{110}{3}\%$

Consider the following for the next two (02) items that follow :

Let $\sin\beta$ be the GM of $\sin\alpha$ and $\cos\alpha$; $\tan\gamma$ be the AM of $\sin\alpha$ and $\cos\alpha$.

31. What is $\cos2\beta$ equal to ?

(a) $(\cos\alpha-\sin\alpha)^2$

(b) $(\cos\alpha+\sin\alpha)^2$

(c) $(\cos\alpha-\sin\alpha)^3$

(d) $\frac{(\cos\alpha-\sin\alpha)^2}{2}$

32. What is the value of $\sec 2\gamma$?

(a) $\frac{3-\sin 2\alpha}{5+2\sin 2\alpha}$

(b) $\frac{5+\sin 2\alpha}{3-\sin 2\alpha}$

(c) $\frac{3-2\sin 2\alpha}{4+\sin 2\alpha}$

(d) $\frac{3-\sin 2\alpha}{4+3\sin 2\alpha}$

Consider the following for the next two (02) items that follow :

A flagstaff 20 m long standing on a pillar 10 m high subtends an angle $\tan^{-1}(0{\cdot}5)$ at a point P on the ground. Let θ be the angle subtended by the pillar at this point P.

33. If x is the distance of P from bottom of the pillar, then consider the following statements :

1. x can take two values which are in the ratio 1 : 3
2. x can be equal to height of the flagstaff

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

34. What is a possible value of $\tan\theta$?

(a) $\frac{3}{4}$

(b) $\frac{2}{3}$

(c) $\frac{1}{3}$

(d) $\frac{1}{4}$

Consider the following for the next two (02) items that follow :

The perimeter of a triangle ABC is 6 times the AM of sine of angles of the triangle. Further $BC=\sqrt{3}$ and $CA=1$.

35. What is the perimeter of the triangle ?

(a) $\sqrt{3}+1$

(b) $\sqrt{3}+2$

(c) $\sqrt{3}+3$

(d) $2\sqrt{3}+1$

36. Consider the following statements :

1. ABC is right angled triangle
2. The angles of the triangle are in AP

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

Consider the following for the next two (02) items that follow :

Let $x = \dfrac{\sin^2 A + \sin A + 1}{\sin A}$ where $0 < A \leq \dfrac{\pi}{2}$

37. What is the minimum value of x ?

(a) 1

(b) 2

(c) 3

(d) 4

38. At what value of A does x attain the minimum value ?

(a) $\dfrac{\pi}{6}$

(b) $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{3}$

(d) $\dfrac{\pi}{2}$

Consider the following for the next two (02) items that follow :

In the triangle ABC,

$a^2 + b^2 + c^2 = ac + \sqrt{3}bc$

39. What is the nature of the triangle ?

(a) Equilateral

(b) Isosceles

(c) Right angled triangle

(d) Scalene but not right angled

40. If $c = 8$, what is the area of the triangle ?

(a) $4\sqrt{3}$

(b) $6\sqrt{3}$

(c) $8\sqrt{3}$

(d) $12\sqrt{3}$

Consider the following for the next two (02) items that follow :

Consider the function

$f(x) = |x-2| + |3-x| + |4-x|$, where $x \in R$.

41. At what value of x does the function attain minimum value ?

(a) 2

(b) 3

(c) 4

(d) 0

42. What is the minimum value of the function ?

(a) 2

(b) 3

(c) 4

(d) 0

Consider the following for the next two (02) items that follow :

Consider the sum

$S = 0! + 1! + 2! + 3! + 4! + \ldots + 100!$

43. If the sum S is divided by 8, what is the remainder ?

(a) 0

(b) 1

(c) 2

(d) Cannot be determined

44. If the sum S is divided by 60, what is the remainder ?

(a) 1

(b) 3

(c) 17

(d) 34

Consider the following for the next two (02) items that follow :

In a triangle PQR, P is the largest angle and $\cos P = \frac{1}{3}$. Further the in-circle of the triangle touches the sides PQ, QR and RP at N, L and M respectively such that the lengths PN, QL and RM are n, $n+2$, $n+4$ respectively where n is an integer.

45. What is the value of n ?

(a) 4

(b) 6

(c) 8

(d) 10

46. What is the length of the smallest side ?

(a) 12

(b) 14

(c) 16

(d) 18

Consider the following for the next two (02) items that follow :

Given that

$\sin x + \cos x + \tan x + \cot x + \sec x + \operatorname{cosec} x = 7$

47. The given equation can be reduced to

(a) $\sin^2 2x - 44 \sin 2x + 36 = 0$

(b) $\sin^2 2x + 44 \sin 2x - 36 = 0$

(c) $\sin^2 2x - 22 \sin 2x + 18 = 0$

(d) $\sin^2 2x + 22 \sin 2x - 18 = 0$

48. If $\sin 2x = a - b\sqrt{c}$, where a and b are natural numbers and c is prime number, then what is the value of $a - b + 2c$?

(a) 0

(b) 14

(c) 21

(d) 28

Consider the following for the next two (02) items that follow :

A quadratic equation is given by

$\left(3+2\sqrt{2}\right)x^2-\left(4+2\sqrt{3}\right)x+\left(8+4\sqrt{3}\right)=0$

49. What is the HM of the roots of the equation ?

(a) 2

(b) 4

(c) $2\sqrt{2}$

(d) $2\sqrt{3}$

50. What is the GM of the roots of the equation ?

(a) $\sqrt{2}\left(\sqrt{6}-\sqrt{3}+\sqrt{2}-1\right)$

(b) $\sqrt{2}\left(\sqrt{6}+\sqrt{3}-\sqrt{2}-1\right)$

(c) $\left(\sqrt{6}-\sqrt{3}+\sqrt{2}-1\right)$

(d) $\left(\sqrt{6}+\sqrt{3}+\sqrt{2}-1\right)$

Consider the following for the next two (02) items that follow :

Let $\Delta(a, b, c, \alpha)=\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$

51. If $\Delta(a, b, c, \alpha)=0$ for every $\alpha > 0$, then which one of the following is correct ?

(a) a, b, c are in AP

(b) a, b, c are in GP

(c) $a, 2b, c$ are in AP

(d) $a, 2b, c$ are in GP

52. If $\Delta(7, 4, 2, \alpha)=0$, then α is a root of which one of the following equations ?

(a) $7x^2+4x+2=0$

(b) $7x^2-4x+2=0$

(c) $7x^2+8x+2=0$

(d) $7x^2-8x+2=0$

Consider the following for the next two (02) items that follow :

Given that $m(\theta)=\cot^2\theta+n^2\tan^2\theta+2n$, where n is a fixed positive real number.

53. What is the least value of $m(\theta)$?

(a) n

(b) $2n$

(c) $3n$

(d) $4n$

54. Under what condition does m attain the least value ?

(a) $n = \tan^2\theta$

(b) $n = \cot^2\theta$

(c) $n = \sin^2\theta$

(d) $n = \cos^2\theta$

Consider the following for the next two (02) items that follow :

A quadrilateral is formed by the lines $x=0$, $y=0$, $x+y=1$ and $6x+y=3$.

55. What is the equation of diagonal through origin ?

(a) $3x + y = 0$

(b) $2x + 3y = 0$

(c) $3x - 2y = 0$

(d) $3x + 2y = 0$

56. What is the equation of other diagonal ?

(a) $x + 2y - 1 = 0$

(b) $x - 2y - 1 = 0$

(c) $2x + y + 1 = 0$

(d) $2x + y - 1 = 0$

Consider the following for the next two (02) items that follow :

$P(x, y)$ is any point on the ellipse $x^2+4y^2=1$. Let E, F be the foci of the ellipse.

57. What is $PE+PF$ equal to ?

(a) 1

(b) 2

(c) 3

(d) 4

58. Consider the following points :

1. $\left(\frac{\sqrt{3}}{2}, 0\right)$

2. $\left(\frac{\sqrt{3}}{2}, \frac{1}{4}\right)$

3. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{4}\right)$

Which of the above points lie on latus rectum of ellipse ?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

Consider the following for the next two (02) items that follow :

The line $y=x$ partitions the circle $(x-a)^2+y^2=a^2$ in two segments.

59. What is the area of minor segment ?

(a) $\frac{(\pi-2)a^2}{4}$

(b) $\frac{(\pi-1)a^2}{4}$

(c) $\frac{(\pi-2)a^2}{2}$

(d) $\frac{(\pi-1)a^2}{2}$

60. What is the area of major segment ?

(a) $\frac{(3\pi-2)a^2}{4}$

(b) $\frac{(3\pi+2)a^2}{4}$

(c) $\frac{(3\pi-2)a^2}{2}$

(d) $\frac{(3\pi+2)a^2}{2}$

Consider the following for the next two (02) items that follow :

Let $A(1, -1, 2)$ and $B(2, 1, -1)$ be the end points of the diameter of the sphere $x^2+y^2+z^2+2ux+2vy+2wz-1=0$.

61. What is $u+v+w$ equal to ?

(a) –2

(b) –1

(c) 1

(d) 2

62. If $P(x, y, z)$ is any point on the sphere, then what is PA^2+PB^2 equal to ?

(a) 15

(b) 14

(c) 13

(d) 6·5

Consider the following for the next two (02) items that follow :

Consider two lines whose direction ratios are (2, –1, 2) and (k, 3, 5). They are inclined at an angle $\frac{\pi}{4}$.

63. What is the value of k ?

(a) 4

(b) 2

(c) 1

(d) –1

64. What are the direction ratios of a line which is perpendicular to both the lines ?

(a) (1, 2, 10)

(b) (–1, –2, 10)

(c) (11, 12, –10)

(d) (11, 2, –10)

Consider the following for the next two (02) items that follow :

Let $\vec{a}=3\hat{i}+3\hat{j}+3\hat{k}$ and $\vec{c}=\hat{j}-\hat{k}$. Let $\vec{b}$ be such that $\vec{a}\cdot\vec{b}=27$ and $\vec{a}\times\vec{b}=\vec{9c}$

65. What is $\vec{b}$ equal to ?

(a) $3\hat{i}+4\hat{j}+2\hat{k}$

(b) $5\hat{i}+2\hat{j}+2\hat{k}$

(c) $5\hat{i}-2\hat{j}+6\hat{k}$

(d) $3\hat{i}+3\hat{j}+4\hat{k}$

66. What is the angle between $(\vec{a}+\vec{b})$ and $\vec{c}$?

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{6}$

Consider the following for the next two (02) items that follow :

Let a vector $\vec{a}=4\hat{i}-8\hat{j}+\hat{k}$ make angles α, β, γ with the positive directions of x, y, z axes respectively.

67. What is $\cos\alpha$ equal to ?

(a) $\frac{1}{3}$

(b) $\frac{4}{9}$

(c) $\frac{5}{9}$

(d) $\frac{2}{3}$

68. What is $\cos2\beta+\cos2\gamma$ equal to ?

(a) $-\frac{32}{81}$

(b) $-\frac{16}{81}$

(c) $\frac{16}{81}$

(d) $\frac{32}{81}$

Consider the following for the next two (02) items that follow :

The position vectors of two points A and B are $\hat{i}-\hat{j}$ and $\hat{j}+\hat{k}$ respectively.

69. Consider the following points :

1. (–1, –3, 1)
2. (–1, 3, 2)
3. (–2, 5, 3)

Which of the above points lie on the line joining A and B ?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

70. What is the magnitude of $\overrightarrow{AB}$?

(a) 2

(b) 3

(c) $\sqrt{6}$

(d) $\sqrt{3}$

Consider the following for the next three (03) items that follow :

Let $f(x) = Pe^x + Qe^{2x} + Re^{3x}$, where P, Q, R are real numbers. Further $f(0) = 6$, $f'(ln\ 3) = 282$ and $\int_0^{ln2} f(x)dx = 11$

71. What is the value of Q ?

(a) 1

(b) 2

(c) 3

(d) 4

72. What is the value of R ?

(a) 1

(b) 2

(c) 3

(d) 4

73. What is $f'(0)$ equal to ?

(a) 18

(b) 16

(c) 15

(d) 14

Consider the following for the next two (02) items that follow :

Suppose E is the differential equation representing family of curves $y^2 = 2cx + 2c\sqrt{c}$ where c is a positive parameter.

74. What is the order of the differential equation ?

(a) 1

(b) 2

(c) 3

(d) 4

75. What is the degree of the differential equation ?

(a) 2

(b) 3

(c) 4

(d) Degree does not exist

Consider the following for the next three (03) items that follow :

Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$

76. What is $f(0)$ equal to ?

(a) -1

(b) 0

(c) 1

(d) 2

77. What is $\lim_{x\to 0}\frac{f(x)}{x}$ equal to ?

(a) -1

(b) 0

(c) 1

(d) 2

78. What is $\lim_{x\to 0}\frac{f(x)}{x^2}$ equal to ?

(a) -1

(b) 0

(c) 1

(d) 2

Consider the following for the next two (02) items that follow :

Let $f(x)=\sin[\pi^2]x+\cos[-\pi^2]x$ where [.] is a greatest integer function

79. What is $f\left(\frac{\pi}{2}\right)$ equal to ?

(a) -1

(b) 0

(c) 1

(d) 2

80. What is $f\left(\frac{\pi}{4}\right)$ equal to ?

(a) $-\frac{1}{\sqrt{2}}$

(b) -1

(c) 1

(d) $\frac{1}{\sqrt{2}}$

Consider the following for the next three (03) items that follow :

Let $I_1=\int_0^{\pi}\frac{x}{1+\cos^2 x}dx$ and

$I_2=\int_0^{\pi}\frac{1}{1+\sin^2 x}dx$

81. What is the value of $\frac{I_1+I_2}{I_1-I_2}$?

(a) 1

(b) π

(c) π^2

(d) $\frac{\pi+1}{\pi-1}$

82. What is the value of $8I_1^2$?

(a) π

(b) π^2

(c) π^3

(d) π^4

83. What is the value of I_2 ?

(a) $\frac{\pi}{\sqrt{2}}$

(b) $\frac{\pi}{2\sqrt{2}}$

(c) $\frac{3\pi}{2\sqrt{2}}$

(d) $\frac{\pi}{4\sqrt{2}}$

Consider the following for the next two (02) items that follow :

Let $I = \int_a^b \frac{|x|}{x} dx$, $a < b$

84. What is I equal to when $a < 0 < b$?

(a) $a + b$

(b) $a - b$

(c) $b - a$

(d) $\frac{(a+b)}{2}$

85. What is I equal to when $a < b < 0$?

(a) $a + b$

(b) $a - b$

(c) $b - a$

(d) $\frac{(a+b)}{2}$

Consider the following for the next three (03) items that follow :

Let $f(x) = |lnx|, x \neq 1$

86. What is the derivative of $f(x)$ at $x = 0{\cdot}5$?

(a) –2

(b) –1

(c) 1

(d) 2

87. What is the derivative of $f(x)$ at $x = 2$?

(a) $-\frac{1}{2}$

(b) –1

(c) $\frac{1}{2}$

(d) 2

88. What is the derivative of $f \circ f(x)$, where $1 < x < 2$?

(a) $\frac{1}{lnx}$

(b) $\frac{1}{xlnx}$

(c) $-\frac{1}{lnx}$

(d) $-\frac{1}{xlnx}$

Consider the following for the next two (02) items that follow :

Let $f(x)=\begin{cases} x+6, & x\leq 1 \\ px+q, & 1<x<2 \\ 5x, & x\geq 2 \end{cases}$

and $f(x)$ is continuous

89. What is the value of p ?

(a) 2

(b) 3

(c) 4

(d) 5

90. What is the value of q ?

(a) 2

(b) 3

(c) 4

(d) 5

91. Consider the following statements :

1. $f(x) = lnx$ is increasing in $(0, \infty)$

2. $g(x) = e^x + e^{\frac{1}{x}}$ is decreasing in $(0, \infty)$

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

92. What is the derivative of $\sin^2 x$ with respect to $\cos^2 x$?

(a) -1

(b) 1

(c) $\sin 2x$

(d) $\cos 2x$

93. For what value of m with $m < 0$, is the area bounded by the lines $y = x$, $y = mx$ and $x = 2$ equal to 3 ?

(a) $-\frac{1}{2}$

(b) -1

(c) $-\frac{3}{2}$

(d) -2

94. What is the derivative of $\text{cosec}(x^\circ)$?

(a) $-\text{cosec}(x^\circ)\cot(x^\circ)$

(b) $-\frac{\pi}{180}\text{cosec}(x^\circ)\cot(x^\circ)$

(c) $\frac{\pi}{180}\text{cosec}(x^\circ)\cot(x^\circ)$

(d) $-\frac{\pi}{180}\text{cosec}(x)\cot(x)$

95. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} = 0$ is

(a) $y = 2x$

(b) $y = 2x + 4$

(c) $y = x^2 - 1$

(d) $y = \frac{(x^2-2)}{2}$

96. If $f(x) = x^2 + 2$ and $g(x) = 2x - 3$, then what is $(fg)(1)$ equal to ?

(a) 3

(b) 1

(c) –2

(d) –3

97. What is the range of the function $f(x) = x + |x|$ if the domain is the set of real numbers ?

(a) $(0, \infty)$

(b) $[0, \infty)$

(c) $(-\infty, \infty)$

(d) $[1, \infty)$

98. If $f(x) = x(4x^2 - 3)$, then what is $f(\sin\theta)$ equal to ?

(a) $-\sin 3\theta$

(b) $-\cos 3\theta$

(c) $\sin 3\theta$

(d) $-\sin 4\theta$

99. What is $\lim_{x \to 5} \frac{5 - x}{|x - 5|}$ equal to ?

(a) –1

(b) 0

(c) 1

(d) Limit does not exist

100. What is $\lim_{x \to 1} \frac{x^9 - 1}{x^3 - 1}$ equal to ?

(a) –1

(b) –3

(c) 3

(d) Limit does not exist

101. The mean and variance of five observations are 14 and 13·2 respectively. Three of the five observations are 11, 16 and 20. What are the other two observations ?

(a) 8 and 15

(b) 9 and 14

(c) 10 and 13

(d) 11 and 12

102. Let A and B be two independent events such that
$P(\overline{A}) = 0{\cdot}7$, $P(\overline{B}) = k$, $P(A \cup B) = 0{\cdot}8$.
What is the value of k ?

(a) $\frac{5}{7}$

(b) $\frac{4}{7}$

(c) $\frac{2}{7}$

(d) $\frac{1}{7}$

103. A biased coin with the probability of getting head equal to $\frac{1}{4}$ is tossed five times. What is the probability of getting tail in all the first four tosses followed by head ?

(a) $\frac{81}{512}$

(b) $\frac{81}{1024}$

(c) $\frac{81}{256}$

(d) $\frac{27}{1024}$

104. A coin is biased so that heads comes up thrice as likely as tails. In four independent tosses of the coin, what is probability of getting exactly three heads ?

(a) $\frac{81}{256}$

(b) $\frac{27}{64}$

(c) $\frac{27}{256}$

(d) $\frac{9}{256}$

105. Let X and Y be two random variables such that $X+Y=100$. If X follows Binomial distribution with parameters $n=100$ and $p=\frac{4}{5}$, what is the variance of Y ?

(a) 1

(b) $\frac{1}{2}$

(c) 16

(d) $\frac{1}{16}$

106. If two lines of regression are $x+4y+1=0$ and $4x+9y+7=0$, then what is the value of x when $y=-3$?

(a) -13

(b) -5

(c) 5

(d) 7

107. The central angles p, q, r and s (in degrees) of four sectors in a Pie Chart satisfy the relation $9p=3q=2r=6s$. What is the value of $4p-q$?

(a) 12

(b) 24

(c) 30

(d) 36

108. The observations 4, 1, 4, 3, 6, 2, 1, 3, 4, 5, 1, 6 are outputs of 12 dices thrown simultaneously. If m and M are means of lowest 8 observations and highest 4 observations respectively, then what is $(2m+M)$ equal to ?

(a) 10

(b) 12

(c) 17

(d) 21

109. A bivariate data set contains only two points (–1, 1) and (3, 2). What will be the line of regression of y on x ?

(a) $x - 4y + 5 = 0$

(b) $3x + 2y - 1 = 0$

(c) $x + 4y + 1 = 0$

(d) $5x - 4y + 1 = 0$

110. A die is thrown 10 times and obtained the following outputs :

1, 2, 1, 1, 2, 1, 4, 6, 5, 4

What will be the mode of data so obtained ?

(a) 6

(b) 4

(c) 2

(d) 1

111. Consider the following frequency distribution :

x	1	2	3	5
f	4	6	9	7

What is the value of median of the distribution ?

(a) 1

(b) 2

(c) 3

(d) 3·5

112. For data –1, 1, 4, 3, 8, 12, 17, 19, 9, 11; if M is the median of first 5 observations and N is the median of last five observations, then what is the value of $4M - N$?

(a) 7

(b) 4

(c) 1

(d) 0

113. Let P, Q, R represent mean, median and mode. If for some distribution $5P = 4Q = \frac{R}{2}$, then what is $\frac{P+Q}{2P+0{\cdot}7R}$ equal to ?

(a) $\frac{1}{12}$

(b) $\frac{1}{7}$

(c) $\frac{2}{9}$

(d) $\frac{1}{4}$

114. If G is the geometric mean of numbers $1, 2, 2^2, 2^3, \ldots, 2^{n-1}$, then what is the value of $1 + 2\log_2 G$?

(a) 1

(b) 4

(c) $n - 1$

(d) n

115. If H is the harmonic mean of numbers $1, 2, 2^2, 2^3, \ldots, 2^{n-1}$, then what is n/H equal to ?

(a) $2-\frac{1}{2^{n+1}}$

(b) $2-\frac{1}{2^{n-1}}$

(c) $2+\frac{1}{2^{n-1}}$

(d) $2-\frac{1}{2^n}$

116. Let P be the median, Q be the mean and R be the mode of observations $x_1, x_2, x_3, \ldots x_n$. Let $S=\sum_{i=1}^{n}(2x_i-a)^2$. S takes minimum value, when a is equal to

(a) P

(b) $\frac{Q}{2}$

(c) $2Q$

(d) R

117. One bag contains 3 white and 2 black balls, another bag contains 2 white and 3 black balls. Two balls are drawn from the first bag and put it into the second bag and then a ball is drawn from the second bag. What is the probability that it is white ?

(a) $\frac{6}{7}$

(b) $\frac{33}{70}$

(c) $\frac{3}{10}$

(d) $\frac{1}{70}$

118. Three dice are thrown. What is the probability that each face shows only multiples of 3 ?

(a) $\frac{1}{9}$

(b) $\frac{1}{18}$

(c) $\frac{1}{27}$

(d) $\frac{1}{3}$

119. What is the probability that the month of December has 5 Sundays ?

(a) 1

(b) $\frac{1}{4}$

(c) $\frac{3}{7}$

(d) $\frac{2}{7}$

120. A natural number n is chosen from the first 50 natural numbers. What is the probability that $n+\frac{50}{n}<50$?

(a) $\frac{23}{25}$

(b) $\frac{47}{50}$

(c) $\frac{24}{25}$

(d) $\frac{49}{50}$